RECTILINEAR DISLOCATION IN AN ANISOTROPIC PLATE

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Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 90–96, 1966

A solution has been found to the problem of calculating the stress and displacement fields caused by a rectilinear dislocation in an anisotropic elastic plate. Special cases of anisotropy have been found with solutions represented by elementary functions.

Certain problems in describing crystal plastic deformation phenomena make it vital to know the fields of the elastic stresses and displacements caused by an individual dislocation in a bounded crystal. It is interesting to study the effect of crystal boundaries on these fields with a simple model which approximates fairly closely to experimental conditions.

The model selected is shown in Fig. 1. A dislocation with a Burgers vector $\mathbf{b}(b_1, b_2, b_3)$ is situated in an infinite elastic anisotropic plate of thickness 2h. The dislocation line is parallel to the plate boundaries. The following restriction is introduced in relation to the plate's elastic properties: the medium has a plane of elastic symmetry perpendicular to the dislocation line. The selection of the coordinate system and position of the dislocation are shown in Fig. 1. The requirement is to find the stresses and displacements at an arbitrary point in the plate.

One limited special form of this problem has been solved by Kroupa [1]. The limitations which he introduced are as follows: the medium is isotropic, the dislocation is at the precise center of the band and the Burgers vector has only one component b_2 differing from zero (the same coordinates were chosen in [1] as in Fig. 1).

Thus Kroupa's results can be obtained from the results of the present work as a special case. Other special cases arising from this problem are those concerning the elastic stress and displacement fields caused by a dislocation in anisotropic semi-bounded [2] and bounded [3] media.

It is immediately apparent that the problem is a plane one, in the sense that the fields to be found do not depend on coordinate z. Since the medium has a plane of elastic symmetry perpendicular to the dislocation line, it is clear from [4] that the system of stresses and strains in such a medium can be divided into two independent subsystems. The first of these is plane deformation with stress components σ_{xx}, σ_{yy} and σ_{xy} differing from zero and displacement vector components u_x and u_y, the second is "antiplane" deformation with stress components σ_{xz} and σ_{yz} differing from zero and the displacement vector component u_z.

In the case under examination, the plane deformation is caused by the Burgers vector edge components b_x and b_y and the antiplane deformation by the screw component b_z. The solution is therefore divided into two stages, corresponding to edge and screw dislocations.

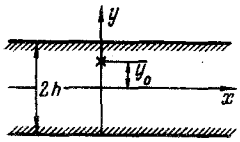

Fig. 1

§1. Edge dislocation in an anisotropic plate. The stress and displacement fields caused by an edge dislocation in an anisotropic plate give rise to a plane problem in anisotropic elasticity theory. For the sake of clarity, we will refer subsequently to the stress field and use the results and nomenclature of S. G. Lekhnitskii [4]. The stress tensor components in a plane anisotropic elasticity problem can be represented as follows:

$$\sigma_{xx} = -2\mathrm{Re}\,[\mu_1^2 f_1(z_1) + \mu_2^2 f_2(z_2)],$$
$$\sigma_{xy} = 2\mathrm{Re}\,[\mu_1 f_1(z_1) + \mu_2 f_2(z_2)],$$
$$\sigma_{yy} = -2\mathrm{Re}\,[f_1(z_1) + f_2(z_2)],$$
$$z_\alpha = x + \mu_\alpha y, \qquad (\alpha = 1, 2). \tag{1.1}$$

Here μ_1 and μ_2 (as well as $\overline{\mu}_1$ and $\overline{\mu}_2$) are the roots of a fourth-degree algebraic equation, the coefficients of which are linked with the elastic modulus tensor components of the medium.

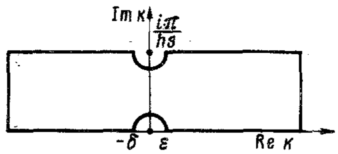

Fig. 2

The displacement vector components are expressed by the functions $f_1(z_1)$, $f_2(z_2)$ and the elastic moduli of the medium [3]. The essence of the problem is to determine the functions $f_1(z_1)$ and $f_2(z_2)$, and to solve it we seek the stresses in the form

$$\sigma_{ik} = \sigma_{ik}{}^\circ + \sigma_{ik}{}^*, \tag{1.2}$$

where the first term corresponds to dislocation in an unbounded medium [3] and the second allows for the presence of bonding surfaces.

The stresses $\sigma_{ik}{}^*$ can be expressed according to (1.1) by the functions $f_\alpha{}^*(z_\alpha)$, which are regular in the strip $-h \le y \le h$.

The boundary conditions in this case consist of the disappearance of stress components σ_{i_2} (i = 1, 2) on the planes $y = \pm h$,

$$\sigma_{22}{}^*(x, h) = -\sigma_{22}{}^\circ(x, h),$$
$$\sigma_{22}{}^*(x, -h) = -\sigma_{22}{}^\circ(x, -h),$$
$$\sigma_{12}{}^*(x, h) = -\sigma_{12}{}^\circ(x, h),$$
$$\sigma_{12}{}^*(x, -h) = -\sigma_{12}{}^\circ(x, -h). \tag{1.3}$$

Thus we arrive at the following problem in the theory of functions of a complex variable: to find the two functions $f_1{}^*(z_1)$ and $f_2{}^*(z_2)$ which are regular in the strip $-h \le y \le h$ and which satisfy (having regard to (1.1)) the given boundary conditions (1.3).

This problem was solved in general form (i.e., with arbitrary fixed loads along the edges) by Kufarev and Sveklo [5]. The limitations imposed on these loads in [5] lie principally in the extent to which they can be represented as Fourier integrals. In addition, the method suggested in [5] was utilized in [6], and gave the solution of the second main problem and of certain problems in anisotropic strip elasticity theory.

Since the Fourier transforms of the boundary loads are elementary functions (see below) in the situation which is of interest to us, it is possible to obtain a solution in expanded form by following the essential features of [5]. For this purpose we examine the Fourier representations of the functions

$$f_j^*(z_j) \quad (j=1,2), \quad f_j^*(z_j) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f_j^*(k)\, e^{-ikz_j} dk. \tag{1.4}$$

Having regard to (1.1), we then obtain from (1.3) a system of equations to find $f_j^*(k)$ (j = 1, 2),

$$f_1^*(k)\, e^{-ik\mu_1 h} + f_2^*(k)\, e^{-ik\mu_2 h} +$$
$$+ \overline{f_1^*(-k)}\, e^{-ik\bar\mu_1 h} + \overline{f_2^*(-k)}\, e^{-ik\bar\mu_2 h} = -P_1(k),$$
$$f_1^*(k)\, e^{ik\mu_1 h} + f_2^*(k)\, e^{ik\mu_2 h} +$$
$$+ \overline{f_1^*(-k)}\, e^{ik\bar\mu_1 h} + \overline{f_2^*(-k)}\, e^{ik\bar\mu_2 h} = -P_2(k),$$
$$\mu_1 f_1^*(k)\, e^{-ik\mu_1 h} + \mu_2 f_2^*(k)\, e^{-ik\mu_2 h} + \bar\mu_1 \overline{f_1^*(-k)}\, e^{-ik\bar\mu_1 h} +$$
$$+ \bar\mu_2 \overline{f_2^*(-k)}\, e^{-ik\bar\mu_2 h} = P_3(k),$$
$$\mu_1 f_1^*(k)\, e^{ik\mu_1 h} + \mu_2 f_2^*(k)\, e^{ik\mu_2 h} + \bar\mu_1 \overline{f_1^*(-k)}\, e^{ik\bar\mu_1 h} +$$
$$+ \bar\mu_2 \overline{f_2^*(-k)}\, e^{ik\bar\mu_2 h} = P_4(k). \tag{1.5}$$

Here $P_i(k)$ (i = 1, 2, 3, 4) are the Fourier transforms of the functions in the right sides of Eqs. (1.3). If we take account of the explicit form of functions $\sigma_{i_2}{}^\circ(x, y)$ (i = 1, 2) [3] and in view of the fact that we select as μ_1 and μ_2 those characteristic equation radicals for which Im $\mu_1 > 0$ and Im $\mu_2 > 0$, we obtain for

$$P_1(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \sigma_{22}{}^\circ(x, h)\, e^{ikx} dx$$

the following expression:

$$P_1(k) =$$
$$= \begin{cases} {}^1\!/_4\, i \sqrt{2/\pi}\, [\lambda_1 e^{ik\mu_1(y_0-h)} + \lambda_2 e^{ik\mu_2(y_0-h)}], & k<0, \\ -{}^1\!/_4\, i \sqrt{2/\pi}\, [\bar\lambda_1 e^{ik\bar\mu_1(y_0-h)} + \bar\lambda_2 e^{ik\bar\mu_2(y_0-h)}], & k>0, \end{cases} \tag{1.6}$$

where $\lambda_j = M_{jk} d_k$ and the values of M_{jk} and d_k are linked with the elastic moduli and the Burgers vector by the well-known relationships given in [3] (summation is represented here and subsequently by twice-repeated Roman indices). The expressions for the remaining Fourier forms of $P_j(k)$ (j = 2, 3, 4) are similar to expression (1.6).

It is apparent from (1.6) and (1.5) that the functions $f_1^*(k)$ and $f_2^*(k)$ have different analytical expressions for k < 0 and k > 0. These expressions will be designated $f_{1,2}^-(k)$ and $f_{1,2}^+(k)$ respectively. The solution for system (1.5) can then be written as follows:

$$f_{1,2}^{\pm}(k) = \pm \frac{i}{2\sqrt{2\pi}} \frac{\Delta_{1,2}^{\pm}(k)}{\Delta(k)} \qquad \left(\pm \frac{i}{2\sqrt{2\pi}} \Delta_{1,2}^{\pm}(k)\right). \tag{1.7}$$

Here $\Delta(k)$ is the determinant for system (1.5), and the expression in brackets represents the determinants obtained from $\Delta(k)$ when the first or second columns are replaced by free terms from Eqs. (1.5).

Thus, following (1.4), we find

$$f_{1,2}^*(z_{1,2}) = -\frac{i}{4\pi} \int_{-\infty}^{0} \frac{\Delta_{1,2}^-}{\Delta} e^{-ikz_{1,2}} dk + \frac{i}{4\pi} \int_{0}^{\infty} \frac{\Delta_{1,2}^+}{\Delta} e^{-ikz_{1,2}} dk. \tag{1.8}$$

For the functions determining the total stress field

$$f_j(z_j) = f_j^\circ(z_j) + f_j^*(z_j), \qquad (j=1,2).$$

Here $f_j^\circ(z_j)$ correspond to a dislocation in an unbounded medium [3], and after elementary conversions the following representations are obtained:

$$f_j(z_j) = \frac{i}{4\pi} \int_{-\infty}^{\infty} \frac{\Delta_j^+}{\Delta} e^{-ikz_j} dk, \quad y > y_0$$
$$f_j(z_j) = -\frac{i}{4\pi} \int_{-\infty}^{\infty} \frac{\Delta_j^-}{\Delta} e^{-ikz_j} dk, \quad y < y_0 \qquad (j=1,2). \tag{1.9}$$

As will be shown below, the integrals in formulas (1.9) converge, so the functions $f_j(z_j)$ determined by these formulas exist and provide a solution to the problem.

§2. Study of integral convergence: some special cases. Examination of the determinant $\Delta(k)$, the integrand denominator in formulas (1.9), will be of vital importance in all subsequent research. This determinant will now be written in expanded form,

$$\Delta(k) = 4\,|\mu_1-\mu_2|^2\,|\mu_1-\bar\mu_2|^2 \left[\frac{\sin kh(\mu_1-\bar\mu_2)\sin kh(\bar\mu_1-\mu_2)}{(\mu_1-\bar\mu_2)(\bar\mu_1-\mu_2)} - \right.$$
$$\left. - \frac{\sin kh(\mu_1-\mu_2)\sin kh(\bar\mu_1-\bar\mu_2)}{(\mu_1-\mu_2)(\bar\mu_1-\bar\mu_2)}\right]. \tag{2.1}$$

If k is regarded as variable in a complex range, $\Delta(k)$ is an integral function having innumerable zeros, only one of which (k = 0) lies on the real axis. Elementary calculations show that k = 0 is a fourth-order zero of function $\Delta(k)$.

The integrand numerators in expressions (1.9) are also integral functions, and it is therefore easy to show that k = 0 will be a first-order zero for these functions.

Thus the integrands in expression (1.9) have but one singular point on the real axis—the coordinate origin, and this singular point is a third-order pole.

In addition, a direct check satisfies us that the integrand in the upper integral of formula (1.9) decreases exponentially as $k \to \pm\infty$ along the real axis, if $y_0 \le y \le h$, and the same is true of the lower integral if $-h \le y \le y_0$. Consequently, these integrals converge at infinity.

Close to the point k = 0 these integrals diverge, both in the ordinary sense and in the sense of the Cauchy principal value. We will now establish the sense in which the convergence of these integrals in the vicinity of zero can be understood. For the sake of clarity we will refer to one of these integrals. The Laurent series for the integrand in (1.9) in the vicinity of zero has the following form:

$$\frac{\gamma_1}{k^3} + \frac{\gamma_2}{k^2} + \frac{\gamma_3}{k} + \ldots, \qquad \gamma_1 \neq 0.$$

If we now integrate our function along the real axis with the discarded interval $(-\delta, \varepsilon)$, $\delta > 0$, $\varepsilon > 0$, linking δ with ε so that

$$\delta = \varepsilon \left(1 + \frac{2\gamma_2}{\gamma_1}\varepsilon\right)^{-1}$$

and then making ε (and thus δ) tend to zero; this ensures convergence of the integral under examination at zero. When δ and ε are linked in this way they are infinitely small; thus convergence of the integrals can be regarded as established, in the sense defined above.

We will now pass on to examine the possibilities of calculating the (1.9) integrals in elementary functions.

Analysis of the dislocation stress field when the functions $f_j(z_j)$ which determine this field are given in integral form (1.9) is extremely difficult. As a result, the basic problem is whether the (1.9) integrals can be expressed in explicit form by elementary functions.

In calculating the integrals it is natural to resort to contour integration in the complex domain, but the first essential for this is to establish the $\Delta(k)$ function zero distribution with complex k's, since these zeros are the integrand poles. Since $\Delta(k)$ has innumerable zeros, when we calculate the integral we obtain a series in which each term corresponds to the residue of the integrand at the corresponding denominator zero.

In general, however, i.e., with aribtrary μ_1 and μ_2, it is impossible even to establish the $\Delta(k)$ zero distribution, not to mention summation of the corresponding series. It is therefore natural to try to find the relationships between parameters μ_1 and μ_2 at which the (1.9) integrals can be calculated in explicit form.

It is clear from [1,7] that in the isotropic situation, which can be obtained quite simply from (1.9) and (1.1) by passing to the limit as $\mu_1 \to i$, $\mu_2 \to i$, the integrals corresponding to (1.9) cannot be taken in elementary functions. A curious feature here is that it is possible to indicate those relationships between μ_1, and μ_2, i.e., those cases of anisotropy, where the evaluation of the integrals is possible.

This occurs if $\Delta(k)$ becomes a periodic function with purely imaginary period; this in turn is possible when parameters μ_1 and μ_2 are purely imaginary: $\mu_1 = is_1$, $\mu_2 = is_2$, and, in addition, the ratio $(s_1 + s_2)/(s_1 - s_2)$ is a rational number. It is easy in these cases to select standard integration contours so that integral evaluation is reduced to calculating the residues at a finite number of integrand poles.

We will illustrate this reasoning with what is perhaps a simpler example, where

$$(s_1 + s_2)/(s_1 - s_2) = 2\,. \tag{2.2}$$

In this case we have from (2.1)

$$\Delta(k) = -16\,(s_1 - s_2)^2 \operatorname{sh}^4 kh\,(s_1 - s_2). \tag{2.3}$$

As is apparent from expanding the appropriate determinants, the integrand numerators have the following form:

$$\operatorname{sh} kh\,(s_1 - s_2)\,[\alpha_1 e^{\beta_1 k} + \ldots + \alpha\, e^{\beta_6 k}].$$

Here α_j and β_j do not depend on k. Thus we find that the calculations are reduced to the evaluation of integrals of the type

$$\int\limits_{-\infty}^{\infty} \frac{e^{\beta k}\, dk}{\operatorname{sh}^3 kh\,(s_1 - s_2)}$$

and this in turn is done by integration along a standard contour, as shown in Fig. 2.

As might be expected, it becomes apparent during the calculation that both the (1.9) integrals represent functions which are analytic continuations of each other.

We present the result of calculating the determining functions $f_j(z_j)$ for (2.2):

$$f_1(z_1) = \frac{1}{32\,(hs)^3}\Big\{\lambda_1\Big[\Big(-ix + \frac{3}{2}s\,(y - y_0)\Big)^2 -$$

$$- 4h^2s^2\Big]\operatorname{cth}\frac{\pi}{2hs}\Big(x + \frac{3}{2}is\,(y - y_0)\Big) +$$

$$+ \lambda_2\Big[\Big(-ix + \frac{1}{2}s\,(3y - y_0)\Big)^2 -$$

$$- h^2s^2\Big]\operatorname{cth}\frac{\pi}{2hs}\Big(x + \frac{1}{2}is\,(3y - y_0) + ihs\Big) -$$

$$- \bar{\lambda}_1\Big[\Big(-ix + \frac{3}{2}s\,(y + y_0)\Big)^2 -$$

$$- h^2s^2\Big]\operatorname{cth}\frac{\pi}{2hs}\Big(x + \frac{3}{2}is\,(y + y_0) + ihs\Big) -$$

$$- \bar{\lambda}_2\Big[-ix + \frac{1}{2}s\,(3y + y_0)\Big]^2\operatorname{cth}\frac{\pi}{2hs}\Big(x + \frac{1}{2}is\,(3y + y_0)\Big)\Big\},$$

$$f_2(z_2) = \frac{1}{32\,(hs)^3}\Big\{3\lambda_1\Big[\Big(-ix + \frac{1}{2}s\,(y -$$

$$- 3y_0)\Big)^2 - h^2s^2\Big]\operatorname{cth}\frac{\pi}{2hs}\Big(x + \frac{1}{2}is\,(y - 3y_0) + ihs\Big) +$$

$$+ \lambda_2\Big[3\Big(-ix + \frac{1}{2}s\,(y - y_0)\Big)^2 +$$

$$+ 4h^2s^2\Big]\operatorname{cth}\frac{\pi}{2hs}\Big(x + \frac{1}{2}is\,(y - y_0)\Big) -$$

$$- 3\bar{\lambda}_1\Big[-ix + \frac{1}{2}s\,(y + 3y_0)\Big]^2\operatorname{cth}\frac{\pi}{2hs}\Big(x + \frac{1}{2}is\,(y + 3y_0)\Big) -$$

$$- \bar{\lambda}_2\Big[3\Big(-ix + \frac{1}{2}s\,(y + y_0)\Big)^2 +$$

$$+ 5h^2s^2\Big]\operatorname{cth}\frac{\pi}{2hs}\Big(x + \frac{1}{2}is\,(y + y_0) + ihs\Big)\Big\}$$

$$s = s_1 - s_2\,. \tag{2.4}$$

It can be directly verified that the stress field determined by means of (2.4) and (1.1) does, in fact, satisfy all the essential conditions, i.e., it has the necessary singularity at the point $(0, y_0)$ and the components σ_{xy} and σ_{yy} disappear if $y = \pm h$. The formulas (2.4), together with (1.1), permit a complete analysis of the edge dislocations in an anisotropic plate if, of course, condition (2.2) is fulfilled. Thus it is easy to establish that as $x \to \pm\infty$ the stresses diminish, generally speaking, as

$$\operatorname{const}\Big(\frac{x}{hs}\Big)^2 \exp\frac{-|\pi x|}{hs}\,.$$

As a characteristic example, we give the results of calculating the interaction force of two dislocations with identical Burgers vectors (0, b, 0) situated at distance x in the median plane. According to Eshelby [8], the force of interaction between two dislocations is determined by the stress field set up by one of them at the point where the other is situated:

$$F_i = \varepsilon_{ikl}\tau_k\sigma_{lm}b_m. \tag{2.5}$$

Here **F** is the force, ε_{ikl} the unit antisymmetric third-order tensor, τ the unit vector of the tangent to the dislocation line, and **b** the Burgers vector. Summation is indicated by twice-repeated indices. In the case under examination, the only component of F_x which differs from zero takes the form

$$F_x = -\frac{bd}{2\,(hs)^3}\,(2x^2 - h^2s^2)\operatorname{csch}\frac{\pi x}{hs} \tag{2.6}$$

where d is a value connected with the Burgers vector and the elastic constants [3]. Since the parameter s is unity in order of magnitude, formula (2.6) shows that the force of interaction between dislocations of the type indicated changes sign at distance $x \sim h$, whereas in unbounded and semi-bounded media similar dislocations repel whatever the distance between them. It would be possible to produce numerous interesting examples of the application of formula (2.4), e.g., calculating the forces of interaction between other types of dislocation, calculating "image" forces, etc., but this is outside the scope of the present work.

To conclude, although formulas (2.4) were obtained by making the artificial assumption (2.2) in relation to the elastic constants of the material, it is to be hoped that in view of the stability of the solutions of the elasticity theory equations in relation to changes in elastic constants, the results based on formulas (2.4) will remain in force for materials with other elastic constants. At least, this conclusion holds good for materials in which the ratio $(s_1 + s_2)/(s_1 - s_2)$ approximates 2.

§3. Screw dislocation in an anisotropic plate. As has been pointed out, a screw dislocation causes antiplane deformation in the plate, and this deformation is characterized by the displacement u_z and the stress components σ_{xz} and σ_{yz}. To find these it is convenient to introduce the function ψ, which is linked to the stress components as follows [9]:

$$\sigma_{i3} = \varepsilon_{3ik} \frac{\partial \psi}{\partial x_k} \qquad (i = 1, 2), \tag{3.1}$$

where ε_{ike} is the unit third-order antisymmetric tensor. The function ψ must satisfy the following equation in a medium containing a screw dislocation with a Burgers vector b at point r_0:

$$\lambda_{ik} \frac{\partial^2 \psi}{\partial x_i \partial x_k} = b\delta(\mathbf{r} - \mathbf{r}_0) \qquad (\lambda_{ik} = \lambda_{i3k3}), \tag{3.2}$$

where λ_{ikem} is the elastic modulus tensor of the medium.

The boundary conditions are the disappearance of stress component σ_{yz} on the planes $y = \pm h$. This condition can be replaced by the condition that function ψ reduces to zero on the region boundary.

The problem is solved by making a linear coordinate transformation according to the relation

$$x_i' = \beta_{ik} x_k \qquad (i = 1, 2), \tag{3.3}$$

with the requirement that the left side of Eq. (3.2) is converted to a Laplacian operator and the initial plate to a plate of the same thickness and orientation. It is easy to produce this transformation in explicit form,

$$\beta_{1i} = \lambda_{i1}^{-1} \sqrt{\Delta}, \quad \beta_{2i}^{-} = \delta_{2i} \quad (\Delta = \lambda_{11}\lambda_{22} - \lambda_{12}^2), \tag{3.4}$$

where Δ is the determinant of the tensor λ_{ik}.

As a result of the conversion by formulas (3.3) and (3.4), Eq. (3.2) appears thus:

$$\frac{\partial^2 \psi}{\partial x'^2} + \frac{\partial^2 \psi}{\partial y'^2} = \frac{b}{\lambda_{22}} \delta(\mathbf{r}' - \mathbf{r}_0'). \tag{3.5}$$

The boundary conditions which function ψ must satisfy remain unchanged.

Equation (3.5) is satisfied by the stress function for a screw dislocation in an isotropic medium, and also by the electrostatic field potential set up by a charged filament in an isotropic medium. With the boundary conditions indicated, this equation is solved by the image method, and the solution, as is already known from [10], has the following form:

$$\psi = \frac{b}{2\pi\lambda_{22}} \operatorname{Re} \ln \frac{\operatorname{ch} \{{}^1/_4 \pi h^{-1} [(x' - x_0') + i(y' - y_0' - 2h)]\}}{\operatorname{ch} \{{}^1/_4 \pi h^{-1} [(x' - x_0') + i(y' + y_0')]\}}. \tag{3.6}$$

The solution of our problem is obtained simply by substituting the old coordinates for the new, according to (3.3) and (3.4). We then obtain

$$\psi = \frac{b}{2\pi\lambda_{22}} \operatorname{Re} \ln \Big[\operatorname{ch} \{{}^1/_4 \pi h^{-1} [\sqrt{\Delta}\, \lambda_{1i}^{-1} (x_i - x_{i0}) + i(y - y_0 - 2h)]\}\Big] \times \Big[\operatorname{ch} \{{}^1/_4 \pi h^{-1} [\sqrt{\Delta}\, \lambda_{1i}^{-1} (x_i - x_{i0}) + i(y + y_0)]\}\Big]^{-1}. \tag{3.7}$$

Formula (3.7) gives the solution to our problem, because formulas for the stress components can be readily obtained by differentiating ψ according to (3.1). In addition, the stress function for a screw dislocation in an unbounded anisotropic medium can be obtained from (3.7) by passing to the limit as $h \to \infty$.

Interpretation of the result is almost the same as for an isotropic plate, that is to say, the resulting field in the plate can be regarded as a field created in an unbounded medium both by the dislocation itself and by its images at points which satisfy the conditions

$$\begin{gathered} {}^1/_4 \pi h^{-1} [\sqrt{\Delta} \lambda_{1i}^{-1} (x_i - x_{i0}) + i(y - y_0 - 2h)] = i({}^1/_2 \pi + k_1 \pi) \\ (k_1 = 0, \pm 1, \pm 2, \pm \ldots), \\ {}^1/_4 \pi h^{-1} [\sqrt{\Delta} \lambda_{1i}^{-1} (x_i - x_{i0}) + i(y + y_0)] = i({}^1/_2 \pi + k_2 \pi) \quad (k_2 = 0, \pm 1, \pm 2 \pm \ldots). \end{gathered} \tag{3.8}$$

It follows from the formulas that the reflected dislocations all lie on the straight line

$$x + \operatorname{tg} \alpha (y - y_0) = 0 \qquad (\operatorname{tg} \alpha = \lambda_{22}/\lambda_{12})$$

which passes through the track of the dislocation line.

In conclusion, I wish to thank A. M. Kosevich for his valuable advice and L. A. Pastur for his constant vigilance and assistance in the work.

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26 April 1965 Khar'kov